PONDICHERRY UNIVERSITY

RAMANUJAN SCHOOL OF MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICS

M.Sc. MATHEMATICS PROGRAMME

SYLLABI

WITH EFFECT FROM THE ACADEMIC YEAR

2017-18

M.Sc. MATHEMATICS

List of Hard Core Courses (to be) offered from the Academic Year 2017-2018

SL. NO.	COURSE CODE	COURSE TITLE
1	MATH-411	Advanced Algebra
2	MATH-412	Real Analysis – I
3	MATH-414	Topology
4	MATH-416	Graph Theory
5	MATH-421	Linear Algebra
6	MATH-422	Lebesgue Measure Theory
7	MATH-423	Complex Analysis
8	MATH-424	Ordinary Differential Equations
9	MATH-425	Real Analysis – II
10	MATH-513	Functional Analysis
11	MATH519	Galois Theory
12	MATH- 537	Partial Differential Equations

SL NO	COURSE CODE	COURSE TITLE
1	MATH 413	Discrete Mathematics
2.	MATH 415	Continuum Mechanics
3.	MATH 417	Scilab Practical
4.	MATH-511	Fluid Mechanics
5.	MATH-512	Differential Geometry
6.	MATH-514	Analytical Dynamics
7.	MATH-515	Fuzzy Sets and its Applications
8.	MATH-516	Number Theory
9.	MATH-522	Algebraic Graph Theory
10.	MATH-523	Algorithmic Graph Theory
11.	MATH-527	Algebraic Number Theory
12.	MATH-528	Advanced Algebraic Number Theory
13.	MATH 529	Theory of Fuzzy sets
14.	MATH-531	Cryptography
15.	MATH-533	Advanced Topics in Topology and Analysis
16.	MATH-534	Approximation Theory
17.	MATH-536	Difference Equations
18.	MATH-538	Lie Groups of Transformations and Differential Equations
19.	MATH-539	Numerical Analysis for Ordinary Differential Equations
20.	MATH-540	Advanced Fluid Mechanics
21.	MATH-541	Integral Equations
22.	MATH-542	Advanced Mathematical Analysis
23.	MATH-544	Elements of Harmonic Analysis
24.	MATH-545	Linear Lie Groups
25.	MATH-547	Advanced Functional Analysis
26.	MATH-549	Commutative Algebra
27.	MATH-551	Functional Analysis- II
28.	MATH-552	Operator Theory
29.	MATH-554	Non-Commutative Rings and Representations
30.	MATH-555	Advanced Complex Analysis
31.	MATH-559	Mathematica Practical
32.	MATH-560	Mathematical Software
33.	MATH-562	Numerical Analysis
34.	MATH-563	Integral Transforms
35.	MATH-564	Discrete Dynamical Systems
36.	MATH-565	Dynamical Systems
37.	MATH-566	Advanced Topology
38.	MATH- 567	Special Functions in Number Theory
39.	MATH- 568	Theory of Partitions
40.	MATH - 570	Introduction to Fuzzy Set
41.	MATH- 571	Calculus of Variations
42.	MATH- 572	Probability and Statistics

M.Sc. Mathematics-List of Soft Core Courses to be offered from the Academic Year 2017 -18

HARD CORE :MATH 411- ADVANCED ALGEBRA (4 credits)

Unit I: Introduction to groups- Dihedral groups- Homomorphism and Isomorphisms- Groups axioms- Subgroups: Definition and Examples - Centralizer and normalizer, Stabilizer and Kernels - Cyclic groups of a subgroup-Subgroup generated by subsets of a group.

Unit II: Quotient groups and Homomorphisms: Definitions and Examples- more on cosets and Lagrange's Theorem- The isomorphism theorems -Transpositions and Alternating groups- Group Actions: Group Actions and Permutation representations-Group acting on themselves by left multiplication-Cayley's theorem.

Unit III: Group acting on themselves by conjugation -The class equation- Automorphisms- The Sylow theorems- The simplicity of A_n .

Unit IV: Direct and semi-direct products and abelian groups: Direct products- The fundamental theorem of finitely generated abelian groups.

Unit V: Ring Homomorphism and quotient rings- properties of ideals-Rings of fractions- The Chinese Remainder theorem- Euclidean domains, Principal ideal domains and Unique factorization domains.

Text Book:

Abstract Algebra (Third Edition) by David S. Dummit and Richard M. Foote, Chapter 1 (Sections 1.2,1.6 and 1.7 only), Chapter 2 (Sections 2.1 to 2.4), Chapter 3 (except 3.4), Chapter 4, Chapter 5 (Sections 5.1 and 5.2 only), Chapter 7 (Section 7.3 to 7.6), chapter 8 and Chapter 10 (Section 10.1 to 10.3).

- 1 M. Artin: Algebra, Prentice-Hall of India, 1991.
- 2. I.N.Herstein: Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
- 3. N.Jacobson: Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
- 4. S.Lang: Algebra, 3rd edition, Addison-Wesley, 1993.

HARD CORE: MATH-412: REAL ANALYSIS – I (4 credits)

Unit-I

Finite, countable and uncountable sets - Metric spaces - Compact sets - Perfect sets - Connected sets - Convergent sequence - Subsequences - Cauchy sequences - Upper and lower limits – Some special sequences.

Unit- II

Series- Series of non- negative terms - The number e - The root and ratio tests - Power series - Summation by parts - Absolute convergence - Addition and multiplication of series - Rearrangements of series.

Unit- III

Limits of functions - Continuous functions - Continuity and compactness - Continuity and connectedness - Discontinuities - Monotonic functions - Infinite limits and limits at infinity.

Unit- IV

The derivative of a real function - Mean value theorems - The continuity of derivatives - L'Hospital's rule - Derivatives of higher order - Taylor's theorem - Derivatives of vector - valued functions.

Unit- V

The Riemann- Stieltjes integral- Definition and existence of the integral - Properties of the integral - Integration and differentiation - Integration of vector- Valued functions - Rectifiable curves.

Text Book

Walter Rudin, Principles of Mathematical Analysis- McGraw Hill International Editions, Mathematics series, 1976 (Chapters 2-6)

- 1. Patrick M. Fitzpatrick, Advanced Calculus, AMS, Pure and Applied Undergraduate Texts, Indian Edition, 2nd edition, 2009.
- 2. Tom Apostol, Mathematical Analysis, Narosa Publishing House, Indian edition, 1985.
- 3. N.L.Carothers, Real Analysis, Cambridge University Press, 2000.
- 4. Karl.R.Stormberg, An Introduction to Classical Real Analysis, AMS Chelsea Publishing, 2015.

HARD CORE: MATH-414: TOPOLOGY (4 Credits)

Unit-I

Revision of sets - Functions - Product of sets - Relations - Countable sets - Uncountable sets - Partially ordered sets and lattices - Metric spaces - Definition and examples - Open sets and closed sets in metric spaces - Open subsets of real line.

Unit -II

Topological spaces -- Definitions and examples - Closure and related concepts – Open bases and open sub bases – Separability and second countability - Lindloff's Theorem

Unit-III

Compactness – Basic results -- Continuous maps on compact sets - Characterization of compactness by basic and sub basic open covers – Tychonoff's theorem - Generalized heine – Borel theorem.

Unit – IV

Compactness for metric spaces – Sequential compactness - Lebesgue covering lemma - Sequential compactness and compactness coincide on metric spaces - T_1 spaces - Hausdorff spaces.

Unit -V

Completely regular spaces and normal spaces – Urysohn's lemma and Tietze extension theorem– Connected spaces – Components of a space.

Text Book

G. F. Simmons, an Introduction to Topology and Modern Analysis, McGraw-Hill Kogakusha, Tokyo, 1963

Chapter 1 – Revision of Sections 1—3, Section 4—8.

- Chapter 2 Sections 9 12
- Chapter 3 Sections 16, 17 and 18
- Chapter 4 Sections 21 24
- Chapter 5 Sections 26 28
- Chapter 6 Sections 31 and 32

- 1. J. R. Munkres, Toplogy, Pearson Edcation Inc., Second Edition, 2000.
- 2. Stephen Willard, General Topology, Dover Publication 2004.
- 3. J. Dugundgi, Toplogy, Allyn and Bacon, Boston, 1966.
- 4. Fred.H.Croom, Principles of Topology, Dover publications, 2016.

HARD CORE: MATH-416 GRAPH THEORY (4 Credits)

Unit -I

Graphs – Subgraphs – Isomorphism of graphs – Degrees of Vertices – Paths and Connectedness – Autumorphism of a Simple Graph – Operations on Graphs – Trees – Centers and Centroid.

Unit -II

Counting the Number of Spanning Trees – Cayley's Formula – Vertex Cuts and Edge Cuts – Connectivity and Edge-connectivity – Blocks – Cyclical Edge-connectivity of a Graph.

Unit -III

Vertex Independent sets and Vertex Coverings – Edge-Independent Sets – Matchings and Factors – M-Augmenting Paths – Matchings in Bipartite Graphs – Halls Theorem on Bipartite graphs – Tutte's 1-Factor Theorem.

Unit –IV

Vertex Coloring – Chromatic Number – Critical Graphs – Brooks' Theorem – Girth – Triangle-Free Graphs – Mycielski's Construction – Edge Colorings of Graphs – Vizing's Theorem – Chromatic Polynomials.

Unit –V

Planar and Nonplanar Graphs – Euler's Formula and its Consequences – K_5 and $K_{3,3}$ are Nonplanar graphs – Dual of a Plane Graph – The Four Color Theorem and the Heawood Five-Color Theorem – Kuratowski's Theorem (without proof).

Text Book:-

1. R. Balakrishnan and K. Ranganathan, A Textbook of Graph Theory (Universitext), Second Edition, Springer New York 2012.

Chapter 1: 1.1-1.6, 1.8 Chapter 3: 3.1-3.5 Chapter 4: 4.1-4.5 Chapter 5: 5.1-5.5 Chapter 7: 7.1,7.2,7.3.1, 7.5,7.6.2,7.9 Chapter 8: 8.1-8.7

- 1. J.A. Bondy and U.S.R. Murty, Graph Theory, Springer 2008.
- 2. Douglas B. West, Introduction to Graph Theory, Second Edition, PHI Learning Private Ltd, New Delhi-2011.
- 3. G. Chartrand, Linda Lesniak and Ping Zhang, Graphs and Digraphs, Fifth Edition, CRC press 2011.

HARD CORE: MATH-421 LINEAR ALGEBRA (4 Credits)

Unit-I

The Algebra of linear transformations-Characteristic roots- Similarity of linear transformations, Invariant subspaces and matrices.

Unit-II

Reduction to triangular forms- Nilpotent transformations - Index of nil potency and invariant of nilpotent transformation.

Unit-III

Jordan blocks and Jordan forms- Modules - Cyclic modules - Fundamental theorem on modules over PID.

Unit-IV

Rational canonical form- Trace- Transpose and Determinants.

Unit-V

Hermitian - Unitary and Normal transformations - Real quadratic forms.

Text Book

I.N.Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

Sections – 6.1, 6.2 and 6.3 Sections – 6.4 and 6.5 Sections – 6.6 and 4.5 Sections - 6.7, 6.8 and 6.9 Sections – 6.10 and 6.11

- 1. M.Artin, Algebra, Prentice-Hall of India, 1991
- 2. N.Jacobson, Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
- 3. S.Lang, Algebra, 3rd edition, Addison-Wesley, 1993
- **4.** P. B. Bhattacharya, S. K. Jain and S.R. Nagpaul, Basic Abstract Algebra (2nd Edition) Cambridge University Press, Indian edition, 1997
- 5. Kenneth Hoffmann and Ray Kunze, Linear Algebra, (Second edition), Pearson, 2015
- 6. S. Friedberg, A.Insel and L.Spence, Linear Algebra, (4th Edition) Pearson, 2015.

HARD CORE: MATH-422 LEBESGUE MEASURE THEORY (4 Credits)

Unit-I

Ring and algebra of sets- σ - algebras- Examples- Algebras and σ - algebras generated by a class of sets - Borel algebra and Borel sets.Lebesgue outer measure on R - Countable sub-additivity - Measurable sets - Examples - σ - algebra structure of measurable sets - Countable additivity of Lebesgue measure on R - Cantor set.

Unit-II

Construction of a non- measurable subset of [0, 1] - Measurable functions- Examples and basic properties - Approximation of measurable and bounded measurable functions by simple measurable functions - Approximation by step functions and continuous functions - Egorov's theorem.

Unit-III

Lusin's theorem - Lebesgue integral of non- negative measurable functions- Integrable functions and Lebesgue integral of integrable functions - Linearity- Monotone Convergence theorem - Fatou's lemma - Dominated convergence theorem - Applications of convergence theorems.

Unit-IV

Comparison of Riemann and Lebesgue integration - Lebesgue integrability of Riemann integrable functions - Characterization of Riemann integrable functions – Improper Riemann integrals and their Lebesgue integrals - Riemann- Lebesgue lemma - Functions of bounded variation - Statement of Vitali's lemma and theorem on almost everywhere differentiability of monotone increasing functions.

Unit-V

Absolutely continuous functions - Examples and properties - Absolute continuity of indefinite integral of Lebesgue integrable functions - Differentiation of indefinite integrals - Characterization of absolutely continuous functions as indefinite integrals.

Text Books

1. H.L.Royden, and P.M.Fitzpatrick, Real Analysis, (Fourth Edition) PHI Learning Private Limited, 2011.

- 1. P.R. Halmos, Measure Theory, Springer, 2nd edition, 1976.
- 2. Walter Rudin, Real and Complex Analysis (Unit 3), Mc Graw-Hill, 1989.
- 3. Inder K. Rana, An Introduction to Measure and Integration(Second Edition), Narosa Publishing House, 2007.
- 4. G de Barra, Measure Theory and Integration(Second Edition), New Age Publishers, 2013.
- 5. N.L.Carothers, Real Analysis, Cambridge University Press, 2000

HARD CORE: MATH 423 COMPLEX ANALYSIS [4-Credits]

Unit I: Analytic functions- Cauchy Riemann equations-Luca's Theorem-Rational functions-Elementary theory of power series- Abel's Limit theorem-Exponential and trigonometric functions-The Logarithm. *[Capter-2, Sections 1,2 and 3]*

Unit II: Conformality-Linear transformations-Cross ratio- Elementary conformal mappings. *[Chapter-3, Sections:2,3 and 4]*

Unit III: Line integrals-Cauchy's theorem for a rectangle- Cauchy's theorem in a disc-Cauchy's integral formula. *[Chapter:4, Sections: 1 and 2]*

Unit IV: Local properties of analytic functions-Zeros and poles- the Maximum principle- The general form of Cauchy's theorem. *[Chapter: 4, Sections:3and 4.]*

Unit V: The Residue Theorem- Argument principle-Evaluation of Definite integrals- Harmonic functions- Poisson's formula- Schwarz's theorem- Taylor series expansion-Laurent's series. *[Chapter: 4, Sections: 5, 6, Chapter: 5, Sections: 1]*

Text Book:

L.V.Ahlfors, Complex analysis, Third Edition, McGraw Hill Book Company, 1979.

References:

- 1. John. B. Conway, Functions of one Complex Variable, Second Edition, Narosa Publishing House, 2002.
- 2. B.C.Palka, An Inroduction to the Complex function Theory, Springer, 1991.
- 3. H.A. Priestley, Introduction to Complex Analysis, Second Edition, Oxford University Press, 2003.
- 4. Donald sarason, Notes on Complex Function Theory, Hindustan Book agency ,1994.

HARD CORE: MATH-424 ORDINARY DIFFERENTIAL EQUATIONS (4 Credits)

Unit-I

Qualitative properties of solutions – The Sturm comparison theorem – Eigen values and Eigen functions and vibrating string.

Unit-II

Series solutions of first order equations – Second order linear equations – Ordinary points - Regular singular points – Gauss Hyper Geometric equations.

Unit-III

Legendre polynomials – Properties of Legendre polynomials – Bessel functions- The Gamma function - Properties of Bessel Function.

Unit-IV

Linear systems – Homogeneous linear system with constant coefficients.

Unit-V

The existence and uniqueness of solutions – The method of successive approximations – Picards's theorem.

Text Book

G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill Book Company, 1972.

Sections: 22-30, 32-35, 37-38 and 55-56.

References

1. Earl Coddington and Norman Levinson, Theory of ordinary Differential equations, TATA McGraw Hill, 2017.

HARD CORE- MATH-425: REAL ANALYSIS – II (4 credits)

Unit-I

(Sections: 7.9 and 7.10 of [3]; Chapter:6 of [2]; Chapter:4 of [4]-Subsections: 4.7-1 & 4.7-2) Improper Riemann integrals - Functions of Bounded variation – Completeness of Metric Spaces - Nowhere dense sets - Baire's Category Theorem.

Unit-II (Chapter: 7 of [1], Subsections 7.1 to 7.25)

Sequence and Series of functions - Examples - Uniform convergence and Continuity -Uniform convergence and Integration - Uniform convergence and Differentiation - Double sequences and series - Iterated limits- Equicontinuous Families of Functions - Arzela – Ascoli Theorem

Unit- III (Chapter: 7 of [1] subsections: 7.26 to 7.33 and chapter 8 of [1])

The Weierstrauss theorem for algebraic polynomials- The Stone - Weierstrauss Theorem - Power Series - The Exponential and Logarithmic Functions - The Trigonometric Functions - Fourier Series - The Weierstrauss theorem for the Trigonometric polynomials.

Unit- IV (Chapter:9 of [1], Subsections: 9.6 to 9.23)

Functions of Several Variables - Linear Transformation - Differentiation - The Contraction Principle.

Unit- V (Chapter: 9 of [1], Subsections: 9.24 to 9.38)

The inverse function Theorem - The implicit Function Theorem - The Rank Theorem - Determinants.

Text Books

- 1. Walter Rudin, Principles of Mathematical Analysis- McGraw Hill International Editions, Mathematics series, 1976.
- 2. Apostol, Mathematical Analysis, Narosa Publishing House, Indian edition, 2002.
- 3. Richard R. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co.1970.
- 4. Erwin Kreyszig, Introductory Functional analysis with Applications, John Wiley, 1989.

- 1. Patrick M. Fitzpatrick Advanced Calculus, Amer. Math. Soc. Pine and Applied Undergraduate Texts, Indian Edition, 2009.
- 2. Kenneth A. Ross, Elementary Analysis, The Theory of Calculus, Springer-Verlag, 1980.
- 3. G.F.Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 2017.
- 4. Ervin Kreyszig, Introductory Functional Analysis with Applications, Wiley, 2007.

HARD CORE- MATH- 513: FUNCTIONAL ANALYSIS (4 Credits)

Unit-I

Normed Linear Spaces – Examples of sequence and function spaces including c_o , l_p and $L_p[a,b]$, for $1 \le p \le \infty$ -Linear transformations –Continuity – Dual Spaces – Product and quotient of normed linear spaces – Completeness of product and quotient of normed linear Spaces – Completeness of the space of all bounded, linear transformations.

Unit- II

Equivalent norms - Completeness of finite dimensional normed linear spaces – Riesz's lemma - Characterization of finite dimensional normed linear spaces as those with compact unit sphere - Continuity of linear maps defined on finite dimensional normed linear spaces.

Unit-III

Hahn-Banach theorem for real vector spaces – Hahn-Banach theorem for real and complex normed linear spaces –Corollaries to Hahn-Banach theorem – The Principle of uniform boundedness – Banach – Steinhauss theorem – Weakly Bounded sets are bounded.

Unit-IV

Closed and open maps – Maps with closed graph – Example of discontinuous, linear map with closed graph – Open mapping theorem and the closed graph theorem – Applications – Inner product spaces – Examples – Inner product spaces and parallelogram law for norm – Orthonormal sets – Bessel's inequality – Gram-Schmidt orthonormalization – Orthonormal basis-Examples.

Unit- V

Separable Hilbert spaces and countable orthonormal basis – Linear isometry onto l_2 - Example of a non-separable Hilbert space – Uncountable orthonormal basis and definition of convergence of Fourier series – Riesz-Fisher's theorem-Orthogonal projections – Closed subspaces are Chebychev - Riesz's representation theorem.

Text Book

M. Thamban Nair, Functional Analysis, Eastern Economy Edition, Prentice~ Hall of India Private Limited, New Delhi (2002).

Reference Books

1. M. Fabian, P. Habala, P. Hakek, V. M Santalucia, J. Pelant and V. Zizler, Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer- 2001

2. B.V.Limaye, Functional Analysis, Wiley Eastern, New Delhi, 1981.

HARD CORE- MATH- 519- GALOIS THEORY (4 credits)

Unit I: Polynomial rings: Definitions and basic properties- Polynomial rings over fields-Polynomial rings that are unique factorization domains -Irreducible criteria- Polynomial rings over fields.

Unit II: Field theory: Basic theory of field extensions-Algebraic Extensions.

Unit III: Splitting fields and Algebraic closures - Separable and inseparable extensions-Cyclotomic polynomials and extensions.

Unit IV: Galois Theory: Basic definitions- The fundamental theorem of Galois Theory - Finite Fields.

Unit V: Composite extension and simple extensions- Cyclotomic extensions and abelian extensions over Q.

Text Book: Abstract Algebra (Second Edition) by David S. Dummit and Richard M. Foote, Chapter 9 (Sections 9.1 to 9.6), Chapter 13 (Section 13.1,13.2 and 13.4 to 13.6), Chapter 14 (Sections 14.1 to 14.4 and 14.6).

- 1 M. Artin: Algebra, Prentice-Hall of India, 1991.
- 2. I.N.Herstein: Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
- 3. N.Jacobson: Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
- 4. S.Lang: Algebra, 3rd edition, Addison-Wesley, 1993.

HARD CORE - MATH-537: PARTIAL DIFFERENTIAL EQUATIONS - (4 Credits)

Unit – I: First Order PDEs

Curves and surfaces- Genesis of first order PDE- Classification of Integrals- Linear equations of first Order- Pfaffian Differential equations- Compatible systems- Solutions of Quasi linear equations

Non-linear first order PDEs : Charpit's method- Special Types of Charpit's- Jacobi's method,-Integral surfaces through a given curve-The Cauchy problem for Quasi Linear case and nonlinear first order PDEs

Unit – II: Second Order PDEs

Genesis of Second order PDEs- Classification of second order PDEs- Canonical forms of Hyperbolic- Elliptic and parabolic type PDEs - Linear PDE with constant coefficients – Method of finding CF and particular integral- Homogeneous linear PDE

Unit – III: Wave Equations

One –dimensional wave equations- Vibrations of a string of Infinite length- Semi-infinite length and finite length- Adjoint Operators, Riemann's Method- Method of separation of variables

Unit – IV: Laplace equations

Derivation of Laplace equations & poisson equation- Boundary value problems- Properties of Harmonic functions- Mean value theorem- Maximum and minimum principles- Separation of variables- Dirichlet problem and Neumann problems for a rectangle and circle

Unit – V: Heat Equations

Heat Conduction Problem in infinite rod case and finite rod case- Duhamel's Principle- Heat conduction equation - Elementary solution- Solution by separation of variables- Classification in n-variables- Families of equi potential surfaces

Text Books

- 1. T. Amarnath, An Elementary Course in Partial Differential Equations, Narosa Publishing House, 2010.
- 2. K. Shankara Rao, Introduction to Partial Differential Equations, PHI Publications, 3rd Edition. 2011.

- 1. I. N. Sneddon, Elements of Partial Differential Equations, McGraw Hill, International Edition, 1986.
- 2. F. John, Partial Differential Equations, Springer Verlag, 1975.
- 3. Lawrence C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, 1998.
- 4. Clive Chester, Techniques in Partial Differential equations, McGraw-Hill Inc., US, 1971.
- 5. Richard Courant and David Hilbert, Methods of Mathematical Physics, Vol-II, Wiley VCH, 1989.

SOFT CORE: MATH-413 DISCRETE MATHEMATICS – (4 Credits)

Unit- I:

Posets and lattices - Lattices as partially ordered sets – Properties of lattices - Lattices as algebraic systems – Sub lattices – Direct product – Homomorphism.

Unit – II:

Special lattices (Complete lattices, Bounded lattices, Complemented lattices, Distributive lattices, Modular lattices) and their properties – Boolean algebra – Switching algebra – Sub algebra – Direct product of Boolean algebra – Boolean homomorphism.

Unit - III:

Join irreduciable elements – Atoms – Stone theorem – Boolean forms and their equivalence – Min terms – Sum of products canonical form – Free Boolean algebra – Max terms and product of sums canonical form – Values of Boolean expressions – Boolean functions – Symmetric Boolean expressions.

Unit – IV:

Logic gates – Combination of gates – Adders – Karnaugh maps – Representation and Minimization of Boolean functions.

Unit –V:

Directed graphs – In and out degrees – Connectedness - – Directed paths and cycles – Moon theorem.

Text Books

- J.P Trembly and R. Manohar: Discrete Mathematical Structures with Applications to Computer Science, Tata McGraw – Hill Publishing Company Ltd, New Delhi 1997. For Units_– I, II, III, IV: Relevant portions of Chapter - 4.
- 2. J.A. Bondy and U.S.R. Murthy: Graph Theory with Applications, Macmillan Press Ltd, New Delhi-1976.

For Unit –V: Relevant portions of Chapter 10.

- 1. J. Johnsonbaurgh, Discrete Mathematics, MacMillan Publishing company, New York 1989.
- 2. R.P. Grimaldi, Discrete and Combinatorial Mathematics (An Applied Introduction), Pearson Edition Asia, New Delhi 2002.
- 3. B. Kolman, R.C. Busby and S.C Ross, Discrete Mathematical Structures, Pearson Editionn Pvt Ltd, New Delhi –2003.

M. Sc. Mathematics SOFT CORE- MATH-415: Continuum Mechanics (4 credits)

UNIT I

Vectors – summation convention – translation and rotation of coordinates – coordinate transformations in general – analytical definitions of scalars, vectors, and Cartesian tensors

UNIT II

Stress stress components –Cauchy's formula – equations of equilibrium – change of stress components in transformation of coordinates – stress components in orthogonal curvilinear coordinates – stress boundary conditions.

UNIT III

Plane rate of stress – Mohr's circle – principal stresses – shearing stresses - stress deviation tensor – Lame's stress ellipsoid.

UNIT IV

Analysis of deformation – strain – strain components in rectangular Cartesian coordinates – infinitesimal rotation – finite strain components.

UNIT V

Derivation of field equations – material description of the motion of a continuum – spatial description – equation of motion – equation of motion - moment of momentum – balance of energy.

Text Book:

1. Y.C.FUNG, "A First Course in Continuum Mechanics "Second Edition, Prentice Hall.

Reference Book:

1. L.A.Segel and G .H.Handelman Mathematics Applied to Continuum Mechanics SIAM (2007)

SOFT CORE: MATH-417 SCILAB PRACTICALS (4 Credits)

UNIT – I

Overview of Scilab - How to get started with Scilab - Getting help from Scilab demonstrations and macros – The Console – The Editor – Batch Processing

Creating Real Variables - Elementary mathematical functions – Booleans – Complex Numbers – Integrers – Floating Points – Strings – Dynamic Variables

UNIT - 2

Matrices – Create Matrices of Real Variables – Accessing Elements of Matirces - Matrices are dynamic – Elementwise Operations Conjugate transpose and non-conjugate transpose - Multiplication of two vectors Comparing two real matrices - Issues with floating point integers - More on elementary functions

- Higher-level linear algebra features

UNIT – 3

Looping and branching - The if, select, for and while statements The break and continue statements Functions - Function libraries - Managing output arguments Levels in the call stack - The return statement - Debugging functions with pause

UNIT - 4

Plotting - 2D plot - Contour plots - Titles, axes and legends - Export

UNIT – 5

Solving Ordinary Differential Equations using Scilab

Text Book:

- 1. Introduction to Scilab Michael Baudin From Scilab Consortiun, 2010 Chapters 1 to 8
- 2. Plotting Using Scilab An open Source Document <u>www.openeering.com</u>

References:

- 1. Modeling and Simulation in Scilab, Stephen L. Campbell, Jean-Philippe Chancelier and Ramine Nikoukhah
- 2. An Introduction to Scilab from a Matlab User's Point of View by Eike Rietsch

SOFT CORE: MATH-511 FLUID MECHANICS (4 Credits)

Unit-I

Equations of motion - Euler's Equation – Conservation of mass – Balance of momentum – Transport theorem - Conservation of energy – Incompressible Flows – Isentropic Fluids – Bernoulli's theorem.

Unit-II

Rotations and vorticity – Kelvin's circulation theorem – Helmboltz's theorem.

Unit-III

Navier- Stokes equations – Scaling properties – Decomposition theorem - Stokes equations – Poiseuille flow .

Unit-IV

Potential flow – Complex potential – Blasius theorem - Kutta-Joukowski theorem – D'Alembert's paradox – Stokes paradox.

Unit-V

Boundary layers – Prandt boundary layer equations –Steady boundary layer flow on a flat plate of infinite width.

Text Book

A. J. Chorin and J. E Marsden, A Mathematical Introduction to Fluid Mechanics, Texts in Applied Mathematics 4, Springer Verlag, 1990.

References:

1. D.J.Acheson, Elementary Fluid Dynamics, Oxford University Press, 1990.

SOFT CORE: MATH 512: DIFFERENTIAL GEOMETRY (4 Credits)

Unit-I

Curves- arc length- Reparametrization-Level curves - Curvature - Plane curves. [Sections: 1.1 to 1.4 and Sections 2.1,2.2.]

Unit-II

Space curves-Torsion- Serret Frenet equations- Simple closed curves- The Isoperimetric Inequality-The Four vertex Theorem. [Sections 2.3 and Sections 3.1 to 3.3.]

Unit-III

Smooth surface- Tangents, normal and orientability- Examples of surfaces- Quadratic surfaces- Triple orthogonal systems- Applications of Inverse function theorem. [Sections 4.1 to 4.7]

Unit-IV

Lengths of curves on surfacesa- First fundamental form- Isometries of surfaces- Conformal mapping of surfaces-Surface area- Equiareal maps and a theorem of Archimedes. [Sections: 5.1 to 5.5]

Unit-V

The Second Fundamental form- The Curvature of curves on a surface- The normal and principal curvature- Euler's theorem- The geometric interpretation of principal curvatures. [Sections: 6.1 to 6.4]

Text Book:

1. Andrew Pressley, *Elementary Differential Geometry*, Springer, 2004.

- 1.
- 1. Christian Bar, *Elementary Differential Geometry*, Cambridge University Press, 2011.
- 2. Thomas F. Banchoff and Stephen T. Lovett, *Differential Geometry of Curves and Surfaces*, A.K Peters/CRC press, 2010.
- 3. W. Klingenberg, A course in Differential Geometry, Springer-Verlag, New York, 1978.

SOFT CORE: MATH-514 ANALYTICAL DYNAMICS (4 Credits)

Unit- I

Generalized coordinates-Virtual displacements-D'Alembert's principle and derivation of the Lagrange equations.

Unit -II

Lagrange equation for non-holonomic constraints-Method of Lagrange multipliers-Velocity dependent potentials-Non-conservative forces and dissipation function- Non-holonomic systems and Lagrange multipliers.

Unit- III

Hamilton's equation-The Hamilton principle - Variational principle- Catenary- Brachistochrone-Derivation of the Hamiltonian equation- Liouville's theorem.

Unit- IV

Canonical transformations – Generating functions- Harmonic Oscillator.

Unit- V

Hamilton – Jacobi theory- Harmonic oscillator- action – Angle variables- Kepler problem- particle motion in a potential with azimuthal symmetry – Slant throw.

Text Book

Walter Greiner, Classical Mechanics, Systems of Particles and Hamiltonian Dynamics, Springer, ISE, 2004.

Unit I: Sections 14-15; Unit II: Sections 16-17; Unit III: Sections 18; Unit IV: Sections 19; Unit V: Sections 20.

- 1. H. Goldstein, Classical Mechanics, Narosa Publishing House, New Delhi, 1985.
- 2. F. Scheck, Mechanics, From Newton's Laws to Deterministic Chaos, Springer, 1999.

SOFT CORE: MATH-515: FUZZY SETS AND ITS APPLICATIONS (4 Credits)

Unit-I Crisp Sets

Basic Definitions - Operations on crisp sets – Properties of crisp set – Crisp relations- Operations on crisp relations – Properties of Crisp relations – Composition of Crisp relations - Characteristic Function-Exercises

Unit – II Fuzzy Sets

Definition of Fuzzy sets - examples - Fuzzy numbers- Characteristics of a Fuzzy Set- Basic operations on fuzzy sets- Properties of Fuzzy sets- Membership functions-Algebraic product and Sum of Fuzzy Sets – Power and related operations on Fuzzy Sets – The extension Principle-Exercise

Unit-III Fuzzy Relations

Definition of Fuzzy relation – Basic operations on Fuzzy relations – Direct product – Projections of a Fuzzy relation – Max-Min and Min-Max compositions – Fuzzy relations and approximate Reasoning – Exercise- Fuzzy relational equation-Problem partitioning – Solution method – Use of Neural network in Fuzzy relation

Unit-IV Fuzzy control systems

Introduction – Fuzzy control structure - Modelling and control parameters – If....and....then rules – Rule evaluation – Conflict resolution – Defuzzification – Fuzzy controller with matrix Representation - Exercises.

Unit-V Applications

Fuzzy Control in Washing Machine – Fuzzy Decision making in forecasting – Fuzzy decision Making in industrial problems – Fuzzy control in traffic control – Fuzzy relational equation in medicine.

Text Books

- 1. George J. Klir/Bo Yuan, Fuzzy Sets and Fuzzy Logic, Prentice Hall of India (2000).
- 2. George Bojadziev and Maria Bojadziev, Fuzzy Sets, Fuzzy Logic, Applications, World Scientific Publishing Co.Pte.Ltd, Singapore, 1996.

- 1. George J. Klir and Tina A. Folger, Fuzzy Sets, Uncertainty and Information, Prentice-Hall of India 1993.
- 2. Witold Pedrycz & Fernando Gomide, An introduction to Fuzzy Set, Prentice-Hall of India, New Delhi.2005.
- 3. James J. Buckely, Esfandiar Eslami, An introduction to Fuzzy Logic and Fuzzy Sets, Springer 2002.
- 4. Abraham Kandel and Gideon Langholz, Fuzzy Control Systems, CRC Press, USA 1994.

SOFT CORE: MATH-516 NUMBER THEORY (4 Credits)

Unit-I

Divisibility: Introduction - Divisibility - Primes.

Unit-II

Congruences - Solution of congruences - Congruences of higher degree - Prime power moduli - Prime modulus - Congruences of degree two, prime modulus - Power residues -Number theory from an algebraic view point Multiplicative groups, rings and fields.

Unit-III

Quadratic reciprocity: Quadratic residues - Quadratic reciprocity - The Jacobi symbol.

Unit-IV

Some functions of number theory: Greatest integer function - Arithmetic functions - The Moebius Inversion Formula - Multiplication of arithmetic functions – Recurrence functions.

Unit-V

Some Diophantine equations: The equation ax+by = c - Positive solutions - Other linear equations - The equation $x^2 + y^2 = z^2$ - The Equation $x^4 + y^4 = z^2$ - Sum of fourth powers - Sum of two squares - The equation $4x^2 + y^2 = n$.

Text Book

Treatment as in Ivan Niven and S. Zuckerman, An Introduction to the Theory of Numbers, John Wiley, New York, 2000.

UnitI: Sections 1.1 - 1.3UnitII: Sections 2.1 - 2.11UnitIII: Sections 3.1 - 3.3UnitIV: Sections 4.1 - 4.5UnitV: Sections 5.1 - 5.6, 5.10 and 5.11

SOFT CORE : MATH-522 ALGEBRAIC GRAPH THEORY (4 Credits)

Unit -I: Linear Algebra in Graph Theory – The Spectrum of a Graph – Characteristic polynomial – Adjacency Algebra - Reduction Formula for χ – Regular Graphs and Line Graphs – Circulant Graph – Spectrum of the Strongly Regular Graph – Cycles and Cuts – The Incidence Matrix – The Laplacian Spectrum.

Unit -II: Spanning Trees and Associated Structures – Kirchhoff's Law – Thomson's Principle – The Tree-Number – A Bound for the Tree Number of Regular Graphs – Determinant Expansions – Elementary Graphs.

Unit -III: Vertex-Partition and the Spectrum – Color Partition – Wilf's Theorem on the Chromatic Number of a Graph – Coloring Problems – The Chromatic Polynomial – Recursive Relation for the Chromatic Polynomial – Quasi-Separable Graphs – Subgraph Expansions – The Rank Polynomial.

Unit -IV: The Multiplicative Expansion – Whitney's Theorem on Counting Subgraphs – The Induced Subgraph Expansion – Baker's Theorem.

Unit -V: The Tutte Polynomial – The λ -operator – The Deletion-Contraction Property – Chromatic Polynomial and Spanning Trees – The Chromatic Invariant.

Text Book:-

1. Norman Biggs, Algbraic Graph Theory, Second Edition, Cambridge University Press, 1993.

- 1. Chris Godsil and Gordon Royle, Algebraic Graph Theory, Springer 2009.
- 2. R. Balakrishnan and K. Ranganathan, A Textbook of Graph Theory (Universitext), Second Edition, Springer New York 2012.

SOFT CORE: MATH - 523 - ALGORITHMIC GRAPH THEORY – (4 Credits)

Unit- I

Graphs and notations – Null, complement and complete graphs – Degrees – Isomorphism – Subgraphs – Paths and cycles – Connectedness - Components – Operations on graphs - Distance, Radius, Diameter, Centers and medians in graphs - Rooted and m ary trees.

Unit – II

Binary trees – and their search – Spanning trees - Search methods: Breadth first search and depth first search algorithms – Introduction to groups, fields and vector spaces – Vector spaces of graphs – Dimensions, Relationships, orthogonality of cycle and cutset subspaces.

Unit – III

Fundamental cycles – Finding all spanning trees of a graph - Cutsets and their properties – Fundamental cutsets – Relation in fundamental cycles and cutsets – On Connectivity and separability – Network flows - (1) isomorphism.

Unit – IV

Incidence matrix and its submatrics – Cycle matrix – Fundamental cycle matrix and its rank and nullity – Cutset and fundamental cutset matrices – Relationship theorem – Path matrix – Adjacency matrix.

Unit –V

The connector problem – Kruskal algorithm – Prim algorithm – The shortest path problem – Dijlstra algorithm – Network models – Flows – cuts – Maximum flow algorithm – The max. flow, min. cut thorem.

Text Books

- 1. K. Thulasiraman and M.N.S. Swamy, Graphs : Theory and Algorithms John Wiley and Sons, Inc., New York (1992).
- 2. Narsingh Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India Ltd (1974).
- 3. J. A. Bondy and U.S.R. Murthy: Graph Theory with Applications, Elsevier Science North Holland (1982).

- 1. G. Chartrand and O. R. Oellermann, Applied and Algorithmic Graph Theory, McGraw Hill New York (1993).
- 2. W. Kocay and D. L. Kreher, Graphs, Algorithms and Optimization, Chapman and Hall CRC Press, London (2005).
- 3. J. Johnsonbaurgh, Discrete Mathematics, Macmillan Publishing Company, New York (1989).

SOFT CORE : MATH-527 ALGEBRAIC NUMBER THEORY (4 Credits)

Unit-I Elementary Number Theory

Integers – Greatest common divisor – Infinitude of primes – Unique factorization in Z – Fermat's little theorem – Euler's Φ function and Euler's theorem – Multiplicative property of Φ function – Applications of unique factorization – The equation $x^2 + y^2 = z^2$ – The equation $x^4 + y^4 = z^2$ – The equation $x^4 - y^4 = z^2$ – Fermat numbers and their properties.

Unit-II Euclidean Rings

Preliminaries: Units, Associates, Irreducible elements, Norm map, Unique factorization domain, Principal ideal domain, Euclidean domain – Gauss' lemma – Gaussian integers – Units and primes in the ring of Gaussian integers – Eisenstein integers – Units in the ring of Eisenstein integers – Factorization of 3 – Order of $Z[\rho]/(\lambda)$.

Unit-III Algebraic Numbers and Integers

Basic concepts – Algebraic number – Algebraic integer – Minimal polynomial Countability of algebraic numbers – Liouville's theorem for R – Algebraic number fields – Theorem of the primitive element –Liouville's theorem for C – Characterization of algebraic integers.

Unit-IV Integral **Bases**

The norm and the trace – Integral basis for an algebraic number field – Algebraic integers of Q ($\sqrt{-5}$) – Existence of an integral basis – Discriminant of an algebraic number field – Index – Determination of an integral basis for the ring of integers of a quadratic number field.

Unit-V Dedekind Domains

Integral closure – Integrally closed ring – Noetherian ring – Dedekind domain – Characterizing Dedekind domains.

Text Book

J.E.Smonde and M.Ram Murty, Problems in Algebraic Number Theory, Graduate Texts in Mathematics, Volume 190, Springer Verlag, New York, 1999. Sections 1.1 and 1.2 Sections 2.1, 2.2 and 2.3 Sections 3.1, 3.2 and 3.3 Sections 4.1, 4.2 and 4.3 Sections 5.1 and 5.2

Reference Books:

1. Pierre Samuel and Allan J Silberger, Algebraic Theory of Numbers, Dover Pub. Inc, 2008.

SOFT CORE: MATH-528 ADVANCED ALGEBRAIC NUMBER THEORY (4 Credits)

Unit-I The Ideal Class Group

Euclidean rings – Hurwitz constant – Fractional ideals – Finiteness of the ideal class group – The class number of an algebraic number field – The class number of $Q(\sqrt{-5})$ – The Diophantine equation $x^2 + 5 = y^3$.

Unit-II Quadratic Reciprocity

Preliminaries – Quadratic residues and quadratic non residues – The Legendre symbol – The quadratic character of -1 and 2 – Gauss sums – The law of quadratic reciprocity.

Unit-III The Structure of Units

Discrete subgroup of \mathbb{R}^m – Dirichlet's unit theorem – Units in real quadratic fields – Pell's equation.

Unit-IV Higher Reciprocity Laws

Cubic reciprocity – Eisenstein reciprocity.

Unit-V Analytic Methods

The Riemann and Dedekind zeta functions – Zeta functions of quadratic fields – Dirichlet's hyperbola method.

Text Book

J. E. Smonde and M. Ram Murty, Problems in Algebraic Number Theory, Graduate Texts in Mathematics, Volume 190, Springer Verlag, New York, 1999.

Sections 6.1, 6.2 and 6.3 Sections 7.1, 7.2 and 7.3 Sections 8.1 and 8.2 Sections 9.1 and 9.2 Sections 10.1 and 10.2

Reference Books:

1. Pierre Samuel and Allan J Silberger, Algebraic Theory of Numbers, Dover Pub. Inc, 2008

SOFT CORE: MATH-529 THEORY OF FUZZY SETS (4 Credits)

Unit-I Fuzzy sets

Basic definitions – Types of fuzzy sets – Basic concepts – α cuts and their properties – Representations of fuzzy sets – first and second decomposition theorems.

Unit-II Operations on fuzzy sets

Types of operations – Fuzzy complements – Fuzzy intersections: t norms – Fuzzy unions: t conorms – Combinations of operations.

Unit-III Elements of fuzzy arithmetic

Fuzzy numbers – Linguistic variables – Arithmetic operations on intervals – Arithmetic operations on fuzzy numbers – Fuzzy equations.

Unit-IV Fuzzy relations

Crisp and fuzzy relations – Projections and cylindric extensions – Binary fuzzy relations – Binary relations on a single set – Fuzzy equivalence relations – Fuzzy compatibility relations.

Unit-V Fuzzy logic

An overview of classical logic – multi valued logics – Fuzzy propositions – Fuzzy quantifiers – Linguistic hedges – Inference from conditional fuzzy propositions.

Text Book

George J. Klir and Bo Yuan, Fuzzy sets and Fuzzy Logic, Theory and Applications,

Prentice - Hall of India Private Limited, New Delhi, 2000.

Reference Book

H. J. Zimmermann, Fuzzy set theory and its Applications, Allied Publishers Limited,

New Delhi,1991.

SOFT CORE: MATH -531 CRYPTOGRAPHY (4 Credits)

Unit I

Introduction: Overview of course- Classical cryptography [parts of Chapter 1].

Unit II

Secret Key Encryption : Perfect Secrecy - One time pads [Chapter 2.1], Stream ciphers and the Data Encryption Standard (DES) [Chapter 3 (excluding 3.6)], The Advanced Encryption Standard (AES) - adopted September 2000.

Unit III

Public Key Encryption : Factoring and the RSA encryption [Chapter 4.1 -4.4], Discrete log- Diffie-Hellman Key Exchange [Chapter 8.4 (only pages 270-273)].

Unit IV

ElGamal encryption [Chapter 5 (only pages 162-164)], Digital Signatures [Chapter 6 (excluding 6.5 - 6.6)], One-time signatures- Rabin and ElGamal signatures schemes- Digital Signature Standard (DSS).

Unit V

Hashing : Motivation and applications- Cryptographically Secure Hashing. [Chapter 7.1-7.3,7.6], Message Authentication Codes (MAC)- HMAC- Network Security - Secure Socket Layer (SSL)- I Psec.,Secret Sharing- Definition. Shamir's threshold scheme [Chapter 11.1], Visual secret sharing schemes.

Text Book

D. R. Stinson, S Cryptography, Theory and Practice, CRC Press, 1995.

Reference Books

1. Richard A. Mollin, An Introduction to Cryptography, Chapman & Hall / CRC, Boca Raton, 2000

2. Dominic Walsh, Codes and Cryptography, Oxford Science Publications, Clarendon Press, Oxford, 1988

SOFT CORE : MATH-533 ADVANCED TOPICS IN TOPOLOGY AND ANALYSIS (4 Credits)

Unit-I

Quotient topology and quotient maps - Examples of quotient spaces - Path connectedness - Standard results - Example of a connected but not path connected space- Locally connected spaces.

Unit-II

The Uryshon's metrization theorem - One point compactification - Stone- Cech compactification - The Arzela - Ascoli theorem.

Unit-III

Local finiteness- Countably locally finite refinement of open coverings of metric spaces – Paracompactness - Standard results - Metric spaces are paracompact.

Unit-IV

Partition of unity - Lp- spaces – Completeness - Dual of Lp[a, b] for $1 \le p \le 1$.

Unit-V

Extreme points - Caratheodory's theorem - Krein- Milman theorem - Milman converse Theorem - Extreme points of the closed unit ball of c, $l\infty$, C(Q), Q compact, Hausdorff and the dual of C(Q) - Stricly convex spaces - Examples.

Text Books:

- 1. James R. Munkres, Topology by James R. Munkres, Pearson, 2nd edition, 2000.
- 2. H.L.Royden, and P.M.Fitzpatrick, Real Analysis, (Fourth Edition) PHI Learning Private Limited, 2011.
- 3. M. Fabian, P.Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizler: Functional Analysis and Infinite dimensional geometry, CMS Books in Mathematics, Springer-Verlag, 2001.

- 1. James Dugundji, General Topology, Allyn and Bacon, Inc.(1966).
- 2. Joseph Conway, A course on Functional Analysis, Springer, 2nd edition, 1997.
- 3. B.V.Limaye, Functional Analysis, Wiley Eastern, New Delhi, 1981

SOFT CORE : MATH-534 APPROXIMATION THEORY (4 Credits)

Unit-I

Interpolation by polynomials - Lagrange interpolation - Vander Monde's determinant-Bernstein polynomials - Weierstrass approximation theorem.

Unit-II

Stone- Weierstrass theorem (Real and complex versions) - Weierstrass theorem as corollary -Approximation of continuous, periodic functions by trigonometric polynomials - Best approximation in C[a, b] with sup norm - Chebychev's Alternation theorem - Theorem of de La Vallee Poussin.

Unit-III

General linear families - Haar system and its characterizations - Uniqueness of polynomials of best approximation - Strong unicity theorem - Harr's unicity theorem.

Unit-IV

An algorithm of Remes and convergence under Haar condition - Strictly convex and uniformly convex Banach spaces - Approximation in inner product spaces – Approximation from closed, convex subsets - Approximation from subspaces of Hilbert spaces - Uniform convexity and continuity of metric projection.

Unit-V

Approximation from finite dimensional subspaces - Normal equations and Gram's determinant - approximation in L2[a, b] - Orthogonal polynomials - Legendre and Chebychev polynomials.

Best approximation by subspaces of Banach spaces - Duality formula - Spaces in which all closed subspaces are proximinal or Chebychev-proximinality of weak* Closed subspaces - Approximation by closed hyperplanes.

Text Book:

1. E.W.Cheney, An Introduction to Approximation theory, McGraw-Hill, 1st edition, 1966.

- 1. B.V. Limaye, Functional Analysis, New Age International pvt. Ltd., 2008.
- 2. Frank Deutsch, Best approximation in inner product spaces, spinger,2001.
- 3. Serge Lang, Real Analysis, Addison- Wesley, 1983.
- 4. Ivan Singer, Best approximation in normed linear spaces by elements of linear Subspaces, Springer-Verlag, 1970.

SOFT CORE : MATH-536 DIFFERENCE EQUATIONS (4 Credits)

Unit-I

The Difference Calculus Definition, Derivation of difference equation- Existence and uniqueness theorem- Operators and E- Elementary difference operators. Factorial polynomials- Operators and the sum calculus- Examples.

Unit-II

First order difference equation General Linear equation- Continued fraction. A general first-order equation – Expansion Techniques.

Unit-III

Linear Difference equations Introduction- Linearly dependent functions-Fundamental theorem for homogeneous equations.

Unit-IV

Inhomogeneous equations In homogeneous equations- Second order equations. Sturn Liouville Difference equations.

Unit-V

Linear Difference equation with constant coefficients introduction-Homogeneous equation- Construction of a difference equation having specified solution- Relationship between Linear difference and differential equation.

Text Book

Ronald E. Mickens, Difference equation - Theory and Application, Chapman & Hall, Second Edition, New York – London, 1990.

Unit 1: Chapter 1: (Sections 1.2-1.8) Unit 2: Chapter 2: (Sections 2.1-2.8) Unit 3: Chapter 3: (Sections 3.1-3.3) Unit 4: Chapter 3: (Sections 3.5-3.6) Unit 5: Chapter 4: (Sections 4.1-4.4)

SOFT CORE: MATH-538 LIE GROUPS OF TRANSFORMATIONS AND DIFFERENTIAL EQUATIONS (4 Credits)

Unit-I

Introduction–Lie groups of transformation – Infinitesimal transformations.

Unit-II

Extended group transformations and infinitesimal transformations (one independent – one dependent and two independent – two dependent).

Unit-III

Lie Algebras and Applications.

Unit-IV

Invariance of first and second order differential equations.

Unit-V

Invariance of partial differential equations of first and second order – elementary examples.

TextBook

Treatment as in G. W. Blueman and S. Kumei, Symmetries and Differential Equations, Springer – Verlag, 1980.

Unit 1 – Chapter 2 (Sections 2.1 - 2.2); Unit 2 – Chapter 2 (Sections 2.3.1 - 2.3.3); Unit 3 – Chapter 2 (Sections 2.4.1 - 2.4.4); Unit 4 – Chapter 3 (Sections 3.1.1 - 3.3.3); Unit 5 – Chapter 4 (Sections 4.4.1 - 4.2.2).

SOFT CORE : MATH-539 NUMERICAL ANALYSIS FOR ORDINARY DIFFERENTIAL EQUATIONS (4 Credits)

Unit-I

Euler's method - Trapezoidal rule - Theta method.

Unit-II

Adams - Bashforth method - Order and convergence - Backward differentiation formula.

Unit-III

Gaussion quadrature - Explicit Runge - Kutta scheme - Implicit Runge Kutta scheme - Collocation.

Unit-IV

Stiff equations - Linear stability domain and A- Stability -- A-stability of RK and multistep methods.

Unit-V

Error Control - Milne Device - Embedded Runge Kutta method.

Text Book

1. Arieh Iserles, A First Course in the Numerical Analysis of Differential Equations, Cambridge University press, 2nd edition, 2008.

Reference Books:

1. Richard L. Burden and J.Douglas faires, Numerical Analysis(9th Edition), Cengage Learning India, 2012.

SOFT CORE : MATH-540 ADVANCED FLUID MECHANICS (4 Credits)

Unit-I

Characteristics - Wave equation - Examples - Riemann invariants - Hodograph transformation - Piston problem.

Unit-II

Shocks - Systems of conservation laws - Weak solution - Rankine - Hugoniot relations - Hugoniot relation - Prandtl's relation - Compressive shocks - Entropy condition.

Unit-III

Riemann problem - Centered waves - Solution of the Riemann problem - Courant – Fricdricts - Lewy condition.

Unit-IV

Combustion waves - Single conservation law - Convex conservation laws - Oleinik's condition – Non convex systems of conservation laws - Solution.

Unit-V

Numerical methods - Finite difference Methods- Forward Difference - Backward Difference - Central difference - Consistency - Order - Stability - Lax's Theorem - Von Neumann Analysis - Godunov scheme - 18 stability - l_2 stability - Lax - Fricdricks scheme - Lax Wendroff scheme - Crank - Nicolson scheme.

Text Books

- 1. Chorin and Marsden, A Mathematical Introduction to Fluid Mechanics, Texts in Applied Mathematics, Springer, Third Edition, 2009.
- 2. A Iserles, A First course in the Numerical Analysis of Differential Equations, Cambridge University Press, 2009.

Reference Books:

1. D.J.Acheson, Elementary Fluid Dynamics, Oxford University Press, 1990

SOFT CORE : MATH-541 INTEGRAL EQUATIONS (4 Credits)

Unit-I

Introduction - Classification of integral equation - examples - IVP for ODE.

Unit-II

BVP for ODE - BVP for elliptic PDE - Abel's problem.

Unit-III

Second order ODE and integral equations -Differential equation theory - initial value problems -Boundary value problems - Singular boundary value problems.

Unit-IV

Integral equations of the second kind - Introduction - Degenerate kernels - a different approach.

Unit-V

Operators - Newmann series.

Text Book

Porter and Stirling, Integral equations, pp 1-94. A practical treatment from spectral theory to applications. - Cambridge: Cambridge University Press, 1996.

SOFT CORE : MATH-542 ADVANCED MATHEMATICAL ANALYSIS (4 Credits)

Unit-I Spaces of functions

Families of functions like periodic functions - Continuous functions- C1- functions- rapidly decreasing functions on Rn which separate points- closed subsets - Partition of unity.

Unit-II Topology on the spaces functions

Uniform convergence - Uniform convergence on compact on polynomials (with emphasis on power series)- Ck-functions- C1-functions on Rn – holomorphic functions on C - Completeness of various spaces of functions under uniform metric-Lp-metric and under uniformly on compact topology.

Unit-III Compact subsets

Arzela - Ascoli theorem - Normal families of holomorphic functions - Hilbert spaces of holomorphic functions - Reproducing kernels.

Unit-IV Fourier analysis

Convolutions - Fourier transform - Approximate identities in L1 (Rn) given by classical kernels like Fejer's kernel.

Unit-V Density

Approximation through convolutions - Density theorems of weierstrass and stone- Korovkin – Density of C1c -functions in Lp.

Text Books

- 1. R. Beals, Advanced mathematical analysis, Springer Verlag, New York, 1973.
- 2. J. B. Conway, Functions of one complex variable, Narosa Publishing House, 1980.
- 3. E. H. Lieb and M. Loss, Analysis, Narosa Book House, New Delhi, 1997.

- 1. W. Rudin, Real and complex analysis, 2nd ed., TMH Edition, 1962.
- 2. K. Yosida, Functional analysis, Springer Verlag, New York, 1968.

SOFT CORE : MATH-544 ELEMENTS OF HARMONIC ANALYSIS (4 Credits)

Unit-I

Basic properties of topological groups, subgroups, quotient groups - Examples of Various matrix groups.

Unit-II

Connected groups - Existence of Haar measure (without proof) - Computation of Haar measure on R, T, Z and some simple matrix groups - Convolution, the Banach algebra L1(G) and convolution with special emphasis on L1(R), L1(T) and L1(Z).

Unit-III

Fourier transform and its properties - Approximate identities in L1(G).

Unit-IV

The Dual group of a locally compact abelian group - Computation of dual groups for R, T, Z.

Unit-V

Classical kernels on R - The Fourier inversion Theorem - Plancherel theorem on R - Plancherel measure on R, T, Z - Discussion on Plancherel theorem on a general locally compact abelian group.

Text Books:

- 1. G. Folland, A course in abstarct harmonic analysis, CRC Press, 1994.
- 2. H. Helson, Harmonic analysis, Trim Series, Hindustan Book Agency, 2nd Edition, 1995.
- 3. Y. Katznelson, Introduction to harmonic analysis, J. Wiley and Sons, 1968.
- 4. L.H. Loomis, An introduction to abstract harmonic analysis, van Nostrand, New York, 1953.

- 1. E. Hewitt & K.A. Ross, Abstract harmonic analysis, Vol. I, Springer Verlag, 1963.
- 2. W. Rudin, Real and complex analysis, Tata Mc Graw Hill, 2nd Edition, 1962.

SOFT CORE : MATH-545 LINEAR LIE GROUPS (4 Credits)

Unit-I

Basic properties of topological groups, subgroups, quotient groups and connected groups.

Unit-II

Linear Lie groups like GL (n,R), GL(n,C), Orthogonal groups, Unitary groups, Motion groups, Heisenberg groups and various properties of them.

Unit-III

Computation of Haar measure for the above groups - The exponential map and the Lie algebras of the above groups.

Unit-IV

Representations of a locally compact group - Adjoint representation - Irreducible representations of SU (2) and SO(3).

Unit-V

Induced representation - Irreducible representations of Motion group M (2) and Heisenberg groups.

Text Books:

- 1. J. L. Clerc, Les repr´esentatios des groupes compacts, Analyse harmonique (J.L.Clerc et al., ed.), C.I.M.P.A., 1982.
- 2. G. Folland, A course in abstract harmonic analysis, CRC Press, 1994.
- 3. S. Kumaresan, A course in differential geometry and lie groups, Trim 22, Hindustan Book Agency, 2002.

Reference Books:

1. . M. Sigiura, Unitary representations and harmonic analysis, An introduction, John - Wiley, 1975.

SOFT CORE : MATH-547 ADVANCED FUNCTIONAL ANALYSIS (4 Credits)

Unit-I

Topological vector spaces-balanced and absorbing sets - Locally convex spaces – Examples-Weak and weak topologies.

Unit-II

Finite dimensional topological vector spaces -Minkowski functional - bounded and totally bounded sets - Metrizable topological vector spaces - Characterization of normable locally convex spaces.

Unit-III

Continuous linear functionals and dual of locally convex spaces - Hahn- Banach separation theorems - Weak topology induced by a subset of the dual polar set – Bipolar theorem.

Unit-IV

Weak* continuous linear functionals on dual normed linear spaces - Goldstein's theorem -Banach -Alaoglu's theorem - Characterization of reflexive spaces as spaces with weakly compact unit balls.

Unit-V

Linear operators – Examples - Integral operators - Inverse and adjoint operators - Adjoint operators in Hilbert spaces - Normal and unitary operators.

Text Books:

- 1. H. H. Schefer, Topological Vector Spaces, Springer-1999.
- 2. M. Fabian, P. Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizle, Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer-Verlag, 2001.
- 3. B.V. Limaye, Functional Analysis, New Age International Pvt. Ltd, 1996.
- 4. Thamban Nair, Functional Analysis, PHI Learning Pvt. Ltd, 2009.

Reference Books:

1. Joseph Conway, A Course on Functional Analysis, Springer- Verlag, 1990

SOFT CORE: MATH-549 COMMUTATIVE ALGEBRA (4 Credits)

Unit-I

Prime ideals- Maximal ideas- Nil radical- Jacobson radical- Operation on ideals- Extension and contraction.

Unit-II

Operation on sub-modules- Direct sum and product- Finitely generated modules- Exact sequences- Tensor product- Restriction and extension of Scalars.

Unit-III

Rings and Modules of Faction and Primary decomposition Local properties extended and contracted Primary decomposition.

Unit-IV

Integral dependence and chain conditions.

Unit -V

Noetherian rings and Artinian rings

Text Book

M. K. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison-Weseley, 1994.

- 1. H. Matsumura, Commutative Ring Theory, Cambridge University Press, 1989.
- 2. I. Kaplansky, Commutavie Rings, University of London press, 1966.
- 3. O. Zariski and P. Samuel, Commutative Algebra, Springer 1976.

SOFT CORE: MATH 551 FUNCTIONAL ANALYSIS – II (4 Credits)

Unit-I Normed linear spaces

Separation theorem and strict separation theorem in normed linear spaces-Applications-Weak and weak* topologies on normed linear spaces, both finite and infinite dimensional

Unit-II Weak and weak* topologies

Conditions for metrizability of weak and weak* topologies on bounded sets-Weak and weak* continuous linear functionals-Separation theorem for spaces with weak or weak* topologies

Unit-III Dual-polar set

Weak topology induced by a subset of the dual-polar set - Bipolar theorem - Goldstein's theorem- Banach - Alaoglu's theorem- Reflexivity and weak convergence.

Unit-IV Operators - I

Linear operators-Examples-Integral operators- Inverse and adjoint operators- Range and null spaces- Adjoint operators in Hilbert spaces- Normal and unitary Operators

Unit-V Operators - II

Compact operators on Banach spaces- Definition, examples and basic properties- Hilbert Schmidt operators

Text Books:

- 1. M. Fabian, P.Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizler, Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer-Verlag, 2001.
- 2. M. Thamban Nair, Functional Analysis A First Course, Prentice-Hall of India Private Ltd, New Delhi, 2002.
- 3. B. V. Limaye, Functional Analysis, New- Age International Pvt. Ltd. 1996.

Reference Books:

1. Joseph Conway, A Course on Functional Analysis, Springer- Verlag, 1990.

SOFT CORE: MATH 552 OPERATOR THEORY (4 Credits)

Unit-I

Banach algebras – Involutive Banach algebras – Various examples including Group algebras – Spectrum – Spectral mapping theorem – Spectral radius formula.

Unit-II

Maximal ideal space for commutative Banach algebras – Gelfand - Naimark theory for commutative Banach algebras – C*- algebras, Examples- Commutative C*- algebras.

Unit-III

Representations of C*- Algebras – Von Neumann's density theorem – Double commutant theorem - GNS constructions.

Unit-IV

Functional calculus – The spectral theorem for normal operators – Spectral theorem for unitary operators – Polar decomposition.

Unit-V

Compact operators – Examples and properties – Spectral theorem for compact operators – Hilbert – Schmidt operators.

Text Books:

1. Sunder, V.S, Functional Analysis – Spectral Theory, Trim Series, Hindustan Book Agency, 1997.

- 1. Takesaki, M, Theory of Operator Algebras I, Springer Verlag, 1979.
- 2. Yosida, K, Functional Analysis, Springer Verlag, 1968.

SOFT CORE: MATH - 554 NON-COMMUTATIVE RINGS AND REPRESENTATIONS (4 Credits)

Unit-I Modules

Modules - Artinian and Notherian modules - Tensor products - Restricted and induced modules - Indecomposable modules – Completely reducible module - Schur Lemma.

Unit-II Radical

Semi simple rings - The radical of a rings - The Jacabson radical – Group algebras - Maschke's Theorem.

Unit-III Structure theory

Structure theory of ring - Density theorem - Widderburn-Artin theorem for semi simple rings.

Unit-IV Representations - I

Representations - linear representation - Matrix representation - Equivalent representation - Invariant subspaces - Irreducible representations

Unit-V Representations - II

Direct sum of representations - Induced representation - restricted representation - Tensor product of representations - Inner products of representation.

Text Book

1. Charles W. Curtis and Irving Reiner, Representation Theory of Finite Groups and Associative Algebras, Inter Science Publishers, 1962. (Chapters 2 and 4).

- 1. William Fulton and Joe Harris, Representation Theory A First Course, Springer International Edition, Springer-Verlag, New York, 2004.
- 2. Jacobson, Basic Algebra II, Hindustan Publishing Corporation (India), 1983.
- 3. I. N Herstein, Non-Commutative Rings, The Mathematical Association of America, 5th Edition, 2005.

SOFT CORE: MATH-555: ADVANCED COMPLEX ANALYSIS (4 Credits)

Unit-I

The space of continuous functions - Spaces of analytic functions – Spaces of meromorphic functions - The Riemann Mapping Theorem.[Chapter-7, Sections: 1,2,3 and 4.]

Unit-II

Weierstrass factorization theorem - Factorization of sine function – The gamma function - The Reimann zeta function. [Chapter-7, Sections: 5,6,7 and 8]

Unit-III

Runge's Theorem Simple Connectedness- Mittag Leeffler's Theorem [Chaper-8]

Unit-IV

Schwarz Reflection Principle - Analytic continuation along a path – Monmodromy theorem. [Chapter-9 Sections1, 2 and 3]

Unit-V

Jensen's formula - The genus and order of an entire function. [Chapter-11, Sections:1,2 and 3]

Text Book:

1. John. B. Conway, Functions of one Complex Variable, Second Edition, Narosa Publishing House, 2002.

- 1. B.C.Palka, An Inroduction to the Complex function Theory, Springer, 1991.
- 2. H.A. Priestley, Introduction to Complex Analysis, Second Edition, Oxford University Press, 2003.
- 3. Donald sarason, Notes on Complex Function Theory, Hindustan Book agency ,1994.
- 4. L.V.Ahlfors, Complex analysis, Third Edition, McGraw Hill Book Company, 1979.

SOFT CORE: MATH-559: Mathematica Practical (4 Credits)

Unit-I

Basic concepts: Constants- Built-In Functions- Basic Arithmetic Operations Strings-Assignment, Replacement, and Logical relations - Loops.

Unit-II

Two dimensional graphics and three dimensional Graphics: Plotting Functions of Single variables and Two Variables - Graphic commands.

Unit- III

Lists: Generating Lists- List Manipulation - Set Theory - Tables and Matrices-Equations- Algebra and Trigonometry- Polynomials.

Unit- IV Differential calculus - Integral calculus - Multivariable calculus

Unit-V

Ordinary Differential Equations - Linear Algebra.

Text Books :

1. Eugene Don, Mathematica, Schaum's Outlines, Tata McGraw-Hill Edition, 2009.

Reference Books:

1. Bruce F. Torrence and Eve A. Torrence, Students Introductions to Mathematica, Cambridge University Press, 2008.

SOFT CORE: MATH: 560 - MATHEMATICAL SOFTWARE (Credits: 4)

Unit-I

LATeX introduction- Installation – Math symbols and tables – TeX symbol and tables – Matrix and lists – Typing Math and text – Text environments.

Unit-II

Document structure – Latex Documents – The AMS articles document class – Bemer Presentation and PDF documents – Long Documents – BibteX – Make index – Books in LateX-Colours and Graphics – TeXCAD – LATeX CAD.

Unit-III

Starting with MATLAB- Variables Vectors, Matrices – Creating Array in MATLAB –Menu, Workspace, working Directory, Command window, Diary, Printing- Built_in function, User defined functions, Script M-files- Complex Arithmatic, Figen values and Eigen vectors – Two and three dimensional Plots.

Unit-IV

Getting around with maple – Maple input and output - Programming in Maple.

Unit-V

Maple: Abstract algebra – Linear algebra – Calculus on numbers – Variables- Complex Arithmatic, Eigen values and Eigen vectors – Two and three dimensional plots.

Text Books

- 1. G. Gratzer, More Math Into LATEX, 4th edition, Springer, (2007).
- 2. AMOS Gilat, MATLAB an introduction with application, WILEY India Edition, (2009).
- 3. Brain R Hunt, Ronald L Lipsman, A Guide to MATLAB for beginners and Experienced users, Cambridge University Press. (2003)
- 4. Ander Heck, Introduction in Maple, Springer, (2007)

SOFT CORE- MATH-562: NUMERICAL ANALYSIS (4 Credits)

Unit-I: Nonlinear Equations in One Variable: Fixed point iterative method – convergence Criterion -Aitken's $\Delta 2$ - process - Sturm sequence method to identify the number of real roots – Newton - Raphson's methods convergence criterion Ramanujan's Method - Bairstow's Method.

Unit-II: Linear and Nonlinear system of Equations:

Gauss eliminations with pivotal strategy jacobi and Gauss Seidel Itervative Methods with convergence criterion. LU - decomposition methods – (Crout's, Choleky and DeLittle methods) – consistency and ill conditioned system of equations - Tri-diagonal system of equations – Thomas algorithm. Iterative methods for Nonlinear system of equations, Newton raphson, Quasi newton and Over relaxation methods for Nonlinear system of equations.

Unit-III: Interpolation:

Lagrange- Hermite- Cubic-spline's (Natural, Not a Knot and Clamped)- with uniqueness and error term, for polynomial interpolation- Bivariate interpolation- Orthogonal polynomials Grams Schmidth Orthogoralization procedure and least square- Chebyshev and Rational function approximation.

Unit -IV: Numerical Integration:

Gaussian quadrature, Gauss-Legendre- Gauss-Chebeshev formulas- Gauss Leguree, Gauss Hermite and Spline intergation – Integration over rectangular and general quadrilateral areas and multiple integration with variable limits.

Unit-V: Numerical solution of ordinary differential equations:

Initial value problems- Picard's and Taylor series methods – Euler's Method- Higher order Taylor methods - Modified Euler's method - Runge Kutta methods of second and fourth order – Multistep method - The Adams - Moulton method - stability - (Convergence and Truncation error for the above methods). Boundary - Value problems – Second order finite difference and cubic spline methods.

Text books

- 1. M. K. Jain, S. R. K. Iyengar and R.K. Jain, Numerical methods for scientific and Engineering computation, Wiley Eastern Ltd. 1993, Third Edition.
- 2. C. F. Gerald and P.O. Wheatley, Applied Numerical Methods, Low- priced edition, Pearson Education Asia 2002, Sixth Edition.
- 3. M. K. Jain, Numerical solution of differential equations, Wiley Eastern (1979), Second Edition.

- 1. S. C. Chapra and P.C. Raymond, Numerical Methods for Engineers, Tata McGraw Hill, New Delhi, 2000
- 2. S. S. Sastry, Introductory methods of Numerical analysis, Prentice Hall of India, New Delhi, 1998.
 - 2. Kendall E.Atkinson, An Introduction to Numerical Analysis(2nd Edition), Wiley, 2008.

SOFT CORE: MATH-563: INTEGRAL TRANSFORMS (4 CREDITS)

Unit- I

Laplace transforms -Important properties- Simple Applications- Asymptotic Properties- Watson's Lemma.

Unit- II

Inversion Integral- The Riemann- Lebesgue Lemma- Dirichlet Integrals- the Inversion- Watson's Lemma for loop integrals- Heaviside series expansion.

Unit- III

Application to ordinary differential equations- Elementary examples- Higher order equations-Partial differential equations- Heat diffusion integral equations.

Unit-IV

Fourier transforms- Exponential- Sine and Cosine transforms- Important properties- Spectral analysis.

Unit- V

Partial differential equations- Potential problems-Water waves-Basic equations-Waves generated by a Surface displacement.

Text Book

1. B. Davies, Integral Transforms and Their Applications, Springer, Texts in Applied Mathematics, 41 Third Edition, 2009.

Reference Books:

1. Ian N. Snedden, The Use of Integral Transforms, McGraw Hill, 1972.

SOFTCORE : MATH-564 DISCRETE DYNAMICAL SYSTEMS (4 Credits)

Unit-I

Orbits - Phase portraits- Periodic points and stable sets. Sarkovskii's theorem

Unit-II

Attracting and repelling periodic points- Differentiability and its implications – Parametrized family of functions and bifurcations- The logistic map.

Unit-III

Symbolic dynamics - Devaney's definition of Chaos - Topological Conjugacy.

Unit-IV

Newton's method-Numerical solutions of differential equations.

Unit-V

The dynamics of Complex functions- The quadratic family and the Mandelbrot set.

Text Book

Richard A. Holmgren, A First Course in Discrete Dynamical Systems, Springer Verlag (1994).

Unit-I [Chapters: 1, 2, 4 and 5], Unit-II [Chapters: 6, 7 and 8], Unit-III [Chapters: 9, 10 and 11], Unit-IV [Chapters: 12 and 13], Unit-V [Chapters 14 and 15].

Reference Books:

1. Robert L.Devaney, A First Course in Chaotic Dynamical Systems, Addison-Wesley Publishing Company, Inc. 1992.

SOFT CORE: MATH: 565 - DYNAMICAL SYSTEMS (4 Credits)

Unit-I (Chapters 1, 2 and 3 of [1])

First order differential equations, The logistic population model, Second order differential equations, planar systems, Planar linear systems, Solving linear systems, The linearity principle, Phase portraits for planar systems.

Unit-II (Chapters: 4 and 6 of [1])

Classification of planar systems, higher dimensional linear systems- Harmonic oscillators -The exponential of a matrix- Nonautonomous linear system.

Unit-III (Chapters 7 and 8 of [1])

Nonlinear systems-Dynamical systems-The existence and uniqueness theorem-continuous dependence of solutions- The variational equation. Equilibiria in nonlinear systems- Nonlinear sinks and sources-saddles and stability-bifurcations.

Unit-IV (Chapters 9 and 10 of [1])

Global nonlinear Techniques- Nullclines -Stability of equilibria – Gradient systems- Hamiltonian systems, closed orbits and limit sets-Local sections and flow boxes- The Poincare Map-Monotone sequences in planar Dynamical systems- The Poincare_ Bendixson theorem-Applications.

Unit-V (Chapter 15 of [1])

Discrete dynamical systems-Bifurcations-The discrete logistic model- Chaos- Symbolic dynamics-The shift map –The Cantor middle_third set.

Text Book:

1. Morris W.Hirsch, Stephen Smale, Robert L.Devaney, Differential Equations and Dynamical systems and An Introduction to Chaos, Second edition, Academic Press(Elsevier) 2004.

- 1. Robert L.Devaney, A First Course in Chaotic Dynamical Systems, Addison-Wesley Publishing Company, Inc. 1992.
- 2. Lawrence Perko, Differential equations and Dynamical Systems, (3rd Edition), Springer 2001.

SOFT CORE : MATH: 566 - ADVANCED TOPOLOGY (4 Credits)

Unit-I (Sections- 25, 29 and 34 of [1])

Connected components- Local connectedness - Locally path connected spaces- Local compactness, One point Compactification, Uryshon Metrization Theorem.

Unit-II (Chapter-10 of [2] and Sections- 22 and 36 of [1]) Note and Filters Ountient topology. Introduction to topological groups.

Nets and Filters- Quotient topology- Introduction to topological groups- Existence of partition of unity- Imbedding theorem for compact m- manifolds.

Unit-III (Sections-38,39,40,41 and 42 of [1])

The Stone -Cech Compactification- Locally finite spaces- Nagata- Smirnov Metrization theorem- Paracompactness- Smirnov Metrization theorem.

Unit-IV (Sections-44 and 50 of [1]) The Peano space-filling curve- Introduction to dimension theory- Imbedding theorem for compact Metrizable spaces.

Unit-V (Sections- 51, 52, 53 and 54 of [1]) Homotopy of paths- The fundamental group- Covering spaces- The fundamental group of the circle.

Text Books:

- 1. James R. Munkres, Topology, Second edition, Pearson Education Inc.,(2002).
- 2. K. D. Joshi, Introduction to General Topology, First edition (revised), New Age International Publishers, 2004.

Reference Books:

1. Stephen Willard, General Topology, Dover, 2004.

SOFT CORE: MATH: 567-SPECIAL FUNCTIONS IN NUMBER THEORY (4 Credits)

Unit I

Introduction to Basic hyper Geometric series- Bionomial theorem- q- bionomial theorem Heine's Trasformation formula- Jackson transformation formula

Unit II

Jacobi's triple product identity and its applications and Quintuple product identity and new identity for $(q;q)^{10}_{\infty}$ with application to partition congruence modulo 11

Unit III

Bilateral Series- Ramanujan I \Box 1summation and related identities- Ramanujan theta function identities involving Lambert series.

Unit IV

q- series and theta functions Entries 18 to 30 Chapter 16 of Ramnujan's notebook.

Unit V

Modular relation related to hypergeometric series and its applications.

Text Book

- 1. Gasper and Rahman, Basic hyper geometric series, Cambridge University press 1990. (Unit I-III)
- 2. BC Berndt Ramanujans notebooks Part II Springer Verlag New York 1991.(Unit IV-V)

SOFT CORE: MATH: 568 - THEORY OF PARTITIONS (4 Credits)

Unit I:

Introduction to basic hyper geometric series, q- binomial theorem, Heine's transformation and Gaussian Polynomial.

Unit II:

Jacobi's triple product identity and its applications, bilateral series and its applications, theta functions.

Unit III:

Partition function and its generating function, Euler theorem for partition.

Unit IV:

Congruence properties of partition functions, Rogers Ramanujan Identities.

Unit IV:

Rank and crank of partitions and restricted partitions.

Text Books

1. Bruce C. Berndt, Number Theory in the Spirit of Ramanujan, AMS (For unit I and II)

2. G. E. Andrews, The Theory of Partitions, Addison Wesley 1979.(For unit III, IV & V)

Reference Books:

1. Gasper and Rahman, Basic hypergeometric Series, Cambridge University Press 1990.

2. G.E Andrews, R. Askey and Ranjan Roy, Special functions, Cambridge University press 2000.

3. Bruce C. Berndt, Ramanujan's Notebooks Vol III, Springer, New York 1991.

SOFT CORE: MATH-570: Introduction to Fuzzy Set (4 credits)

Unit 1 Basics on Fuzzy Set

Crispness-Vagueness-Fuzziness-Uncertainty-Fuzzy Set Theory- -Theoretic Operations for Fuzzy Set- Types of Fuzzy Sets- Operations on Fuzzy Set- Algebraic Operations- Set-Theoretic Operations.

Unit 2 Generalization of crisp mathematical concepts to fuzzy sets

Operations for Type2 Fuzzy Sets- Algebraic Operations with Fuzzy Numbers- Special Extended Operations- Extended Addition- Extended Product- Extended Subtraction- Extended Division- Extended Operations for LR-Representation of Fuzzy Sets.

Unit 3 Fuzzy Analysis

Fuzzy Functions on Fuzzy Sets-Integration of Fuzzy Functions-Integration of a Fuzzy Function over a Crisp Interval-Some Properties of Integrals of Fuzzy Functions-Integration of a (Crisp) Real-Valued Function over Fuzzy Interval-Fuzzy Differentiation.

Unit 4 Fuzzy Control

Origin and Objective-The Fuzzy Controller-Types of Fuzzy Controllers-The Mamdani Controller-Defuzzification-The Sugeno Controller- Design Parameters-Scaling Factors-Fuzzy Sets-Rules-Adaptive Fuzzy Control.

Unit 5 Fuzzy Data Analysis

Methods for Fuzzy Data Analysis-Algorithmic Approaches- Knowledge-Based Approaches-Dynamic Fuzzy Data Analysis- Similarity of Functions.

Text Book:

H.J. Zimmermann: Fuzzy set theory and its Applications, Springer Science + Business Media New York, 2001.

Reference Books:

George J. Klir and Bo Yuan: Fuzzy sets and Fuzzy Logic: Theory and Applications, Prentice – Hall of India Private Limited, New Delhi, 2000.

George Bojadziev and Maria Bojadziev, Fuzzy Sets, Fuzzy Logic, Applications, World Scientific Publishing Co.Pte.Ltd, Singapore, 1995.

SOFT CORE: MATH-571 -CALCULUS OF VARIATIONS [4 Credits]

Unit I: Functionals- some simple variational problems-The variation of a functional- A necessary condition for an extremum- The simplest variational problem-Euler's equation-The case of several variables-A simple variable end point problem- The variational derivative-Invariance of Euler's equation. *[Chapter-1]*

Unit II: The fixed end point problem for n-unknown functions - Variational problem in parametric form- Functionals depending on higher order derivatives-Variational problems with subsidiary conditions. [Chapter-2]

Unit III: The general variational of a functional- derivation of the basic formula- End points lying on two given curves or surfaces- Broken extremals- The Weierstrass Erdmann conditions. *[Chapter-3]*

Unit IV: The canonical form of Euler equations- First integrals of the Euler equations- The Legendre transformation- Canonical transformations- Noether's Theorem- The principle of least action- Conservation laws- The Hamilton Jacobi equation- Jacobi theorem. *[Chapter-4]*

Unit V: The second variation of a functional- The formula for the second variation, Legendre conditions- Sufficient conditions for a weak extremum. *[Chapter-5]*

Text Book:

I.M. Gelfand and S.V.Fomin, Calculus of Variations, Dover Publications, 2000.

- 1. A.S. Gupta, Calculus of Variations with Applications, Prentice-Hall of India, 2008.
- 2. M.L.Krasnov, G.I.Makarenko and A.I. Kiselev, *Problems and Exercises in the Calculus of Variations*, Mir Publishers, Moscow 1975.

SOFT CORE: - MATH-572: Probability and Statistics [4 Credits]

Unit I: The probability set function – Random variables – Probability density function – Distribution function – Mathematical expectation – Special mathematical expectations – Chebyshev's Inequality – Conditional probability – Marginal and conditional distributions – Stochastic independence. [Chapters 1 and 2 (except 1.1 and 1.2) of the text book]

Unit II: Some special distributions: The Binomial and related distributions – The Poisson distribution – The Gamma and Chi-Square Distributions – The Normal distribution- The Bivariate normal distribution. [*Chapter -3 of the text book*]

Unit III: Distributions of functions of random variables - Sampling theory – Transformations of variables of the discrete type – Transformations of variables of the continuous type – The b, t and F distributions- Distributions of order statistics- The moment generating function technique. [Chapter 4 [sections 4.1 to 4.7] of the text book.]

Unit IV: The distributions of \overline{X} and nS^2/σ^2 - Expectations of functions of random variables – Limiting distributions: Limiting moment generating functions – The Central limit theorem. [*Chapter-4* [sections 4.8 and 4.9] and *Chapter-5 of the text book.*]

Unit V: Introduction to statistical inference: Point Estimation – Confidence intervals for means – Confidence intervals for differences of means - Confidence intervals for variances. *[Chapter-6 of the text book]*

Text Book:

Robert V. Hogg and Allen T. Craig, *Introduction to Mathematical Statistics* (Fifth Edition) Pearson Education, 2005.

- 1. Paul L.Meyar, *Introductory to Probability and Statistical Applications*, Oxford&IBH Publishers Co. Pvt .Ltd, 1969.
- 2. Arnold Naiman, Gene Zirkel and Robert Rosenfield, *Understanding Statistics*, McGraw-Hill, 1986.
- 3. William Feller, An Introduction to Probability Theory and its Applications, Vol.I, John Wiley, Third Edition, 2008.
- 4. A.Mood, F.Graybill, and D.Boes, *Introduction to the Theory of Statistics*, Tata McGraw Hill (Third Edition) 2008.